VARIANCES IN METAL ACCOUNTING

LBMA Assaying and Refining Conference

Peter Gaylard

20 March 2017
Overview

• Introduction and background
• Metal Accounting
• Reconciliation of Metal Accounting Figures
• Definitions
• Variances
• Metal Reconciliation Example
• Conclusions
Introduction and Background

• Talk based on the development of the AMIRA Code of Practice for Metal Accounting
  ➢ Sponsored by a number of the world’s largest mining companies
  ➢ Released in 2007 and now used in many operations around the world
  ➢ Project also generated a text book “An Introduction to Metal Balancing and Reconciliation”, published by the University of Queensland in 2008
  ➢ Need to acknowledge the work of my colleagues:
    ➢ Neville Randolph (sampling and analysis), Mike Wortley (mass measurement) and Dr Rob Morrison (Editor of the textbook)
• Since release of the Code the ‘Team’ have been asked to conduct numerous metal accounting audits at a variety of operations
• Have also given a number of training workshops, initially at sponsors’ operations, but more recently at various training venues, including several in-house courses
• February 2016: one-day workshop on Metal Accounting presented at Rand Refinery
  ➢ Four months later invited me to be their guest at this conference and to speak on “Variances”
    ▪ Had to try to remember what it was I had said.
  ➢ LBMA correspondence proposed “Variances in Metal Accounting”
  ➢ Conference Program circulated in February: “Metal Reconciliation”
• Will try to touch on all three
Each figure used for Metal Accounting, whether feed, product (concentrate, matte or final metal), or discard, is based on:

1. Measurement of the mass of the material concerned
2. Analysis of a sample of the material
   - Sample obtained by sampling of the bulk material
   - Primary sample may require splitting to give secondary or laboratory sample
   - Laboratory sample will require preparation and splitting to give analytical sample
     - Preparation could involve moisture determination
     - Crushing or lump breaking
     - Blending
   - At each of these process steps there is the possibility of error
     - Contamination, loss of sample (eg dust), sample segregation

- In Metal Accounting there tends to be a strong focus on the performance of the analytical laboratories involved and the effects of sampling and mass measurement errors tend to be overlooked
Reconciliation of Metal Accounting Figures

• Reconciliation of metal accounting figures is required whenever a commercial exchange of metal-bearing material (ore, concentrate, matte or final metal) occurs, resulting in a custody transfer
  ➢ The receiver has to be satisfied that he is not paying too much for the metal content, while the sender / seller has to be satisfied that he is not being under paid
• Hence there is a risk associated with each such transaction
• Reconciliation is achieved when each party agrees that the metal accounting figures recorded separately by them are both accurate and are within acceptable limits of precision
  ➢ This requires that the parties understand the possible sources of error in the figures being reconciled and their prior agreement on what those acceptable limits of precision are
Definitions: Accuracy and Precision

- **Accuracy** – A measurement is accurate if it, or the average of a number of measurements, is close to the true value. In metallurgical operations this true value is unknown.

- **Precision** – Of a measurement depends on the closeness of the outcomes of a repeated measurement or test procedure. Hence it depends only on the distribution of random errors and not on any relationship to a “true” value.
  
  - Usually expressed as the **standard deviation** of the estimated quantity
  - Alternatively, as the **relative standard deviation**: the standard deviation expressed as a percentage of the mean of the measurements of that value.
DEFINITIONS 2

• **Mean (of data)** – The mean of a set of n items of data is the arithmetic average of the series of measurements \( x_i \). The mean is usually designated by \( x \) with a bar above it.

\[
\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{x_1 + x_2 + x_3 \cdots x_n}{n}
\]

**Variance**: the sum of the squares of the differences between each measured value and the mean of the measured values, divided by one less than the number of measurements in the data set

\[
s^2 = \left[ (x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 \cdots (x_n - \bar{x})^2 \right]/(n-1) = \frac{1}{(n-1)} \sum_{i=1}^{n} (x_i - \bar{x})^2
\]

The “**standard deviation**” \( s \) or \( sd \) is then defined as the square root of the Variance
Variances

• In Metal Accounting, variances occur where any measurement of material is carried out, or where that material is divided (sampled)
  ➢ Mass measurement: \( V_M \)
  ➢ Moisture determination: \( V_{H2O} \)
  ➢ Sampling: \( V_S \)
  ➢ Sample Preparation: \( V_{SP} \)
  ➢ Analysis: \( V_A \)

• Variances are additive
  ➢ Total Metal Accounting Variance:
    \[
    V_{MA} = V_M + V_{H2O} + V_S + V_{SP} + V_A \quad \text{Equation A}
    \]
Metal Reconciliation Example: PGM Toll Smelting

• The following PGM toll smelting example is based on PGM industry experience and was included in a paper presented at a SAIMM conference ("Platinum 2012")
• Small platinum producer producing 120,000 ounces per year of platinum from Merensky Reef
• Typically, such a producer would ship 80 – 90 tonnes per day of concentrate in three 30-tonne truck loads
• Each truck weighed and sampled by both the shipper and receiver
• Each day’s receipts constitute an accounting batch
Toll Treatment Terms

• Standard commercial terms based on receiver’s mass and moisture measurements and sample.
  ➢ Sample split and analysed by both receiver and shipper
  ➢ Analytical results exchanged on a pre-arranged date
  ➢ If results do not agree within specified limits, settlement delayed until final metal values are agreed by negotiation or by umpire analysis

• Shipper entitled to be present at weighing and sampling of concentrate at receiver’s plant

• Shipper should also have similar sampling and mass measurement facility to identify possible biases or errors
## Effect of Splitting Limits on Daily Shipments

<table>
<thead>
<tr>
<th>Metal</th>
<th>Daily Shipment (oz or t)</th>
<th>Metal Prices ($ per oz)</th>
<th>Value Shipped ($)</th>
<th>Splitting Limits (%)</th>
<th>Risk to Each Party ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONC., t</td>
<td>90</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pt</td>
<td>367</td>
<td>1000</td>
<td>367 000</td>
<td>6</td>
<td>11 010</td>
</tr>
<tr>
<td>Pd</td>
<td>163</td>
<td>760</td>
<td>123 880</td>
<td>6</td>
<td>3 716</td>
</tr>
<tr>
<td>Au</td>
<td>23</td>
<td>1230</td>
<td>28290</td>
<td>12</td>
<td>1 697</td>
</tr>
<tr>
<td>Rh</td>
<td>25</td>
<td>920</td>
<td>23 000</td>
<td>7</td>
<td>805</td>
</tr>
<tr>
<td>Ru</td>
<td>39</td>
<td>110</td>
<td>4 290</td>
<td>10</td>
<td>215</td>
</tr>
<tr>
<td>Ir</td>
<td>3</td>
<td>1100</td>
<td>3 300</td>
<td>12</td>
<td>198</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>620</strong></td>
<td><strong>549 760</strong></td>
<td></td>
<td></td>
<td><strong>17 641</strong></td>
</tr>
</tbody>
</table>
These are defined in the toll treatment commercial terms and define the acceptable limit of analytical difference between the receiver’s and the shipper’s laboratory for each valuable metal.

The figures quoted are not atypical of actual industrial performance and reflect the accepted level of error in the concentrate analyses for each reported metal.

On a daily shipment with a value of $549 760, there is a total potential error of $35282, or 6.4%.

In this case, both reporting laboratories analyse the same sample and the receiver’s mass is used.

The precision levels shown here are only those related to each laboratory:
- Splitting the laboratory sample to give the analytical sample
- Analysis of the sample

The precision levels related to mass measurement and sampling are not taken into account in assessing the risk to each of the parties.

Are these additional sources of variances significant?
Typical Precision Levels

• From industrial experience, typical precision levels for the different components of the determination of the metal content of each shipment are:

  ➢ Mass: ±0.5%
  ➢ Moisture: ±1%
  ➢ Sampling / Sample Preparation: ±5%
  ➢ Analysis, as shown: ±6.4%
Applying typical precision levels, for mass and moisture measurement and sampling, to the total 620 ounces of PGM in the shipment gives the following:

<table>
<thead>
<tr>
<th>Source of Error</th>
<th>Typical Precision</th>
<th>Error Ounces</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>± 0,5%</td>
<td>3,10</td>
<td>$V_M$</td>
</tr>
<tr>
<td>Moisture</td>
<td>± 1%</td>
<td>6,20</td>
<td>$V_{H_2O}$</td>
</tr>
<tr>
<td>Sampling</td>
<td>± 5%</td>
<td>31,00</td>
<td>$V_S$</td>
</tr>
<tr>
<td>Analysis</td>
<td>± 6,4%</td>
<td>39,68</td>
<td>$V_A$</td>
</tr>
<tr>
<td>TOTAL</td>
<td>± 8,2%</td>
<td>√$V_{MA}$</td>
<td>50,82</td>
</tr>
</tbody>
</table>

- Thus, the overall error in each shipment is ± 8,2%
- The total value of this error is $45080 and the risk to each party is $22540 and not $17641 as indicated from the analytical splitting limits
• From this analysis, one might conclude that since the overall risk has risen from 6.4% to 8.2% there is no need to pay much attention to the mass measurement and sampling errors, until the analytical errors can be reduced.
• Equation A can be re-written as \[ V_{MA} = V_{M/H_2O} + V_{S/SP} + V_A \]
• The “Tripod of Measurements”: these should all have similar values: if one is out of line, the tripod tilts.
• In this case, \[ V_{M/H_2O} \ll V_A ; V_{S/SP} < V_A \]
• However, the total risk has risen from $35282 to $45080, an increase of 28%, which is not insignificant.
• This is a simplified approach for illustrative purposes:
  ➢ A more rigorous approach: carry out the same exercise for each metal.
# Full Variance Analysis

<table>
<thead>
<tr>
<th>Metal</th>
<th>Pt</th>
<th>Pd</th>
<th>Au</th>
<th>Rh</th>
<th>Ru</th>
<th>Ir</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily oz</td>
<td>367</td>
<td>163</td>
<td>23</td>
<td>25</td>
<td>39</td>
<td>3</td>
</tr>
<tr>
<td>Source</td>
<td>Error, %</td>
<td>Error Oz</td>
<td>Variance</td>
<td>Error Oz</td>
<td>Variance</td>
<td>Error Oz</td>
</tr>
<tr>
<td>Mass</td>
<td>0.5</td>
<td>1.835</td>
<td>3.367</td>
<td>0.815</td>
<td>0.664</td>
<td>0.115</td>
</tr>
<tr>
<td>Moisture</td>
<td>1</td>
<td>3.67</td>
<td>13.469</td>
<td>1.63</td>
<td>2.660</td>
<td>0.23</td>
</tr>
<tr>
<td>Sampling</td>
<td>5</td>
<td>18.35</td>
<td>336.722</td>
<td>8.15</td>
<td>66.423</td>
<td>1.15</td>
</tr>
<tr>
<td>Analysis</td>
<td>6</td>
<td>22.02</td>
<td>484.880</td>
<td>9.78</td>
<td>95.648</td>
<td>2.76</td>
</tr>
<tr>
<td>Total Variance, $s^2$</td>
<td>838.438</td>
<td>165.395</td>
<td>9.0069</td>
<td>4.701</td>
<td>19.202</td>
<td>0.152725</td>
</tr>
<tr>
<td>SD</td>
<td>28.955</td>
<td>12.787</td>
<td>3.001</td>
<td>2.168</td>
<td>4.382</td>
<td>0.391</td>
</tr>
<tr>
<td>Rel SD, %</td>
<td>7.9</td>
<td>7.8</td>
<td>13.04</td>
<td>8.7</td>
<td>11.2</td>
<td>13</td>
</tr>
<tr>
<td>Risk, $</td>
<td>28933</td>
<td>9663</td>
<td>3691</td>
<td>2001</td>
<td>480</td>
<td>429</td>
</tr>
<tr>
<td>Analytical Risk, $</td>
<td>22020</td>
<td>7432</td>
<td>3394</td>
<td>1610</td>
<td>430</td>
<td>396</td>
</tr>
<tr>
<td>Effect of Additional Variances, $</td>
<td>6913</td>
<td>2231</td>
<td>297</td>
<td>391</td>
<td>50</td>
<td>33</td>
</tr>
<tr>
<td>Effect of Additional Variances, %</td>
<td>31.4</td>
<td>30.0</td>
<td>8.8</td>
<td>24.3</td>
<td>11.6</td>
<td>8.3</td>
</tr>
</tbody>
</table>

Delivering solutions through collaboration
Conclusions

1. The increase in the total overall risk determined by the full analysis is only $117, so the simplistic approach was not wrong.

2. However, it highlights the effect on the major sources of financial risk, namely Pt, Pd and Rh where the increase in the risk was between 24.6% for Rh and 31.4% for Pt.

3. These three metals effectively account for the overall 28% increase in the total risk.

4. The total risk could, therefore, be significantly reduced through improvement in the sampling precision, in particular.
   - In Metal Accounting there tends to be a strong focus on the performance of the analytical laboratories involved and the effects of sampling and mass measurement errors tend to be overlooked.
   - Determining the variance for each source of error enables those that are excessive to be identified and, where appropriate and where costs permit, corrective action can be taken.
Thanks

• I have already acknowledged my colleagues in this work

• I must thank Rand Refinery Limited and the LBMA for inviting me to be here, for sponsoring my visit here, and for giving me the privilege of attending this conference.